Assignment 2 220246

Task 3: Binomial Pricing and Tree Model (Theoretical)

1. The stock of a company is currently quoted in the market at ₹150. The

price of the stock is expected to go up or down by 10% in next one year

and by 15% in the second year. The risk-free interest rate in the economy is 6%.

Required: Using two-step Binomial Model, find out the price of a 2-year American

put option on the company's stock with strike price of ₹ 170. (Hint: Option Premium cannot be

greater than Intrinsic value)

Given:

- Current stock price (S0) = ₹150

- Expected annual increase or decrease in stock price (u) = 10%

- Expected annual decrease in stock price (d) = -10%

- Risk-free interest rate (r) = 6%

- Strike price (K) = ₹170

- Time periods (t) = 2 years

Step 1: Calculating u and d

- u = 1.10

- d = 0.90

Step 2: Calculating the risk-neutral probability (p)

- p = (e^(r \* Δt) - d) / (u - d)

- p = (e^(0.06 \* 1) - 0.90) / (1.10 - 0.90)

- p ≈ (1.0618 - 0.90) / 0.20

- p ≈ 0.1618 / 0.20

- p ≈ 0.809

Step 3: Constructing the binomial tree for stock prices

- Constructing a binomial tree with two time steps and calculate the stock prices at each node.

Step 4: Calculating option payoffs at each node

- At each node, we'll calculate the option payoff using the formula: Payoff = max(0, K - S)

- At the final nodes:

- Stock price at node (up, up) = S0 \* u^2 = ₹150 \* (1.10)^2 ≈ ₹181.50

- Stock price at node (up, down) = S0 \* u \* d = ₹150 \* 1.10 \* 0.90 ≈ ₹148.50

- Stock price at node (down, up) = S0 \* u \* d = ₹150 \* 1.10 \* 0.90 ≈ ₹148.50

- Stock price at node (down, down) = S0 \* d^2 = ₹150 \* (0.90)^2 ≈ ₹135.00

- Option payoff at each final node:

- Payoff (up, up) = max(0, K - S) = max(0, ₹170 - ₹181.50) = ₹0

- Payoff (up, down) = max(0, K - S) = max(0, ₹170 - ₹148.50) = ₹21.50

- Payoff (down, up) = max(0, K - S) = max(0, ₹170 - ₹148.50) = ₹21.50

- Payoff (down, down) = max(0, K - S) = max(0, ₹170 - ₹135.00) = ₹35.00

Step 5: Backtracking through the tree to calculate option prices

- Starting from the second time step, and backtracking through the tree to calculate the option prices at each node, considering early exercise for American options.

- At each node, the option price is the discounted expected value of the option payoffs in the next period.

- Option price at node (up, up) = e^(-r \* Δt) \* [p \* Option payoff (up, up) + (1 - p) \* Option payoff (up, down)] = e^(-0.06 \* 1) \* [0.809 \* ₹0 + (1 - 0.809) \* ₹21.50] ≈ ₹11.35

- Option price at node (up, down) = e^(-r \* Δt) \* [p \* Option payoff (down, up) + (1 - p) \* Option payoff (down, down)] = e^(-0.06 \* 1) \* [0.809 \* ₹21.50 + (1 - 0.809) \* ₹35.00] ≈ ₹24.29

- Option price at node (down, up) = e^(-r \* Δt) \* [p \* Option payoff (down, up) + (1 - p) \* Option payoff (down, down)] = e^(-0.06 \* 1) \* [0.809 \* ₹21.50 + (1 - 0.809) \* ₹35.00] ≈ ₹24.29

Step 6: Determine early exercise decision for American options

- For American options, compare the option payoff with the option price calculated in the previous step and choose the higher value at each node.

Step 7: Finding the option premium

- The option premium is the option price at the initial node.

- Option premium ≈ ₹24.29

Final Answer:

The price of the 2-year American put option on the company's stock with a strike price of ₹170 is approximately ₹24.29.

2. Consider a two-year call option with strike price Rs. 50 on a stock the current of which is also

Rs. 50. Assume that there are 2 time periods of 1 year and in each year the stock price can move

up or down by an equal percentage of 20%. The risk-free interest rate is 6%. Using binomial option

model, calculate the probability of price moving up and down. Also draw a two-step binomial tree

showing prices and payoffs at each node.

Solution:

**Calculate the Up and Down Factors**

Given that the stock price can move up or down by 20%:

* Up factor (𝑢*u*) = 1 + 0.20 = 1.20
* Down factor (𝑑*d*) = 1 - 0.20 = 0.80

**Calculate the Risk-Neutral Probabilities**

The risk-neutral probability 𝑝*p* for an upward movement is given by: 𝑝=𝑒𝑟Δ𝑡−𝑑𝑢−𝑑*p*=*u*−*der*Δ*t*−*d*​

Where:

* 𝑟*r* is the risk-free interest rate (0.06 or 6%)
* Δ𝑡Δ*t* is the length of the time period (1 year)

Plugging in the values: 𝑝=𝑒0.06⋅1−0.801.20−0.80=𝑒0.06−0.800.40*p*=1.20−0.80*e*0.06⋅1−0.80​=0.40*e*0.06−0.80​

First, calculate 𝑒0.06*e*0.06: 𝑒0.06≈1.06184*e*0.06≈1.06184

Then: 𝑝=1.06184−0.800.40≈0.261840.40≈0.6546*p*=0.401.06184−0.80​≈0.400.26184​≈0.6546

The probability of the stock price moving down (1−𝑝1−*p*) is: 1−𝑝≈1−0.6546=0.34541−*p*≈1−0.6546=0.3454

**Construct the Binomial Tree**

We start with the current stock price of Rs. 50 and apply the up and down factors for each time period.

**Stock Prices**

* At 𝑡=0*t*=0: 𝑆0=50*S*0​=50
* At 𝑡=1*t*=1:
  + 𝑆1𝑢=𝑆0⋅𝑢=50⋅1.20=60*S*1*u*​=*S*0​⋅*u*=50⋅1.20=60
  + 𝑆1𝑑=𝑆0⋅𝑑=50⋅0.80=40*S*1*d*​=*S*0​⋅*d*=50⋅0.80=40
* At 𝑡=2*t*=2:
  + 𝑆2𝑢𝑢=𝑆1𝑢⋅𝑢=60⋅1.20=72*S*2*uu*​=*S*1*u*​⋅*u*=60⋅1.20=72
  + 𝑆2𝑢𝑑=𝑆2𝑑𝑢=𝑆1𝑢⋅𝑑=60⋅0.80=48*S*2*ud*​=*S*2*du*​=*S*1*u*​⋅*d*=60⋅0.80=48
  + 𝑆2𝑑𝑑=𝑆1𝑑⋅𝑑=40⋅0.80=32*S*2*dd*​=*S*1*d*​⋅*d*=40⋅0.80=32

The two-step binomial tree for stock prices is

72

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60 48

/ \ / \

50 48 32

72 / \ 60 / \ / \ 50 48 32

**Payoffs at Maturity (t = 2)**

The call option payoffs at 𝑡=2*t*=2 with a strike price of Rs. 50 are:

* At 𝑆2𝑢𝑢=72*S*2*uu*​=72: max⁡(72−50,0)=22max(72−50,0)=22
* At 𝑆2𝑢𝑑=48*S*2*ud*​=48: max⁡(48−50,0)=0max(48−50,0)=0
* At 𝑆2𝑑𝑢=48*S*2*du*​=48: max⁡(48−50,0)=0max(48−50,0)=0
* At 𝑆2𝑑𝑑=32*S*2*dd*​=32: max⁡(32−50,0)=0max(32−50,0)=0

The payoffs are:

22

/ \

0 0

/ \ / \

50 48 32

22 / \ 0 0 / \ / \ 50 48 32

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